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Direct Adaptive and Neural Control of Wing-Rock Motion of Slender Delta Wings

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The question of wing-rock suppression of slender delta wings is considered. Based on a nonlinear model, an adaptive control law for wing-rock control is derived. In this derivation, it is assumed that the aerodynamic parameters in the nonlinear model are not known. The derivation of a control law for wing-rock suppression using neural networks when the dynamics of wing rock are completely unknown is also treated. A radial basis function network is used for synthesizing the controller. An adaptation law is derived for adjusting the parameters of the network. Digital simulation results show that in the closed-loop system wing-rock motion is suppressed using the adaptive and neural controllers.

Nomenclature

A_m	= reference model matrix
b	= chord
C_l	= roll-moment coefficient
c_i, σ_i	= center and width of Gaussian function
I	= identity matrix
I_{xx}	= mass moment of inertia
P, Q	= matrices in Lyapunov equation
S	= wing reference area
U_∞	= freestream velocity
u	= control input
V	= Lyapunov function
w	= neural network weight vector
$\tilde{w}, \tilde{\theta}, e$	= parameter and tracking errors
x, x_m	= state vectors of system and reference model
θ	= adaptation gain vector
$\lambda_{\min}(Q)$	= minimum eigenvalue of Q
ρ	= density of air
ψ_i	= Gaussian function

I. Introduction

THE phenomenon of wing rock is manifested by limit cycle oscillation in roll, which in some cases is coupled with yaw oscillation. Recently, several theoretical and experimental studies have been performed to understand the dynamics of wing rock and to predict the amplitude and frequency of oscillation of the limit cycle.[1–4]

Nonlinear aerodynamic mathematical models of swept slender wings for one and three degrees of freedom for calculating wing-rock characteristics have been developed in Ref. 1. Analytical models of the wing-rock phenomenon have been developed to predict roll divergence, periods, and amplitudes of oscillations in Refs. 2 and 3. It has been shown in Refs. 4 and 5 that at high angles of attack strong vortices emanating from the forebody of an aircraft are primarily responsible for wing rock in this flight regime. Wind tunnel tests performed on a highly slender forebody and a 78-deg swept delta wing have confirmed that the main flow phenomena causing the wing rock are the interactions between the forebody and the wing vortices.[6] In a recent interesting paper, Luo and Lan[7] considered the question of control of wing-rock motion. Using Beecham and Titchener's[8] averaging technique, an optimal control input is derived in this paper. Since the control input is a function of time, an approximate state-variable feedback control input is obtained using the least-squares method. The aerodynamic rolling moment is a complex nonlinear function of the roll angle, roll rate, angle of attack, and sideslip angle. Controller design in Ref. 7 is based on a nonlinear model in which the aerodynamic parameters appear linearly. For the derivation of the controller, it is assumed that these aerodynamic parameters are known. In a practical situation, such an assumption is not valid. In fact, although different models of wing-rock motion have been proposed, these models are only approximate, and in fact, the exact analytical expression of the rolling moment is difficult to derive.

Neural networks have architecture that has massively parallel interconnections of simple "neural" processors. These networks have ability to "learn" a mapping between an input space and an output space. Recent results in the approximation of nonlinear functions using neural networks have generated considerable interest, and researchers have focused attention on identification and control of dynamic systems using neural networks.[9–12] Recently, radial basis function (RBF) neural networks have been used for the identification of an aircraft (F-16) pitch dynamics and an indirect inverse adaptive controller has been presented.[13] In Ref. 14, a multilayer neural network has been used for the prediction of the force and moment coefficients and for the maneuver of a 70-deg sweep delta wing at high angle of attack.

We are interested in the design of control systems for the wing-rock control under different assumptions related to the knowledge of the roll dynamics. Two kinds of models are assumed for the design: 1) a linearly parameterized nonlinear model and 2) a model with unknown dynamics. We assume for the first model that the aerodynamic coefficients are unknown but the structure of the nonlinear function is known. The second model does not assume any knowledge of the structure of the rolling moment.

Contribution of this paper lies in the design of an adaptive control system and a neural controller for the wing-rock control. For the linearly parameterized model with unknown parameters, a control law is derived using adaptive control techniques of Ref. 15. For the model with unknown dynamics, a RBF neural network is used in the feedback path for control. Based on the Lyapunov stability theory and adaptive control methodology, an adaptation law is derived for continuously adjusting the weights of the network. In the closed-loop system, wing-rock control is accomplished using each of the controllers. For a slender delta wing, numerical results are presented to show that in the closed-loop system the adaptive and neural control systems accomplish wing-rock control in spite of the uncertainty in the roll dynamics.

II. Wing-Rock Dynamics and Control

In this study, we consider a nonlinear mathematical model for wing rock developed by Hsu and Lan[1] and Elzebda et al.[3] The

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differential equation describing the wing-rock motion is given by[2,7]

$$\ddot{\phi} = \left(\rho U_\infty^2 Sb/2I_{xx}\right)C_l + d_0 u \tag{1}$$

where ϕ is the roll angle and an overdot denotes a derivative with respect to time t. Here, for generality, we have introduced the control effectiveness element d_0 relating the input signal u and the angular acceleration. Following Refs. 1 and 3, the roll-moment coefficient is written as

$$C_l = a_0 + a_1\phi + a_2\dot{\phi} + a_3|\phi|\dot{\phi} + a_4|\dot{\phi}|\dot{\phi} + a_5\phi^3 \tag{2}$$

The aerodynamic parameters a_i are nonlinear functions of the angle of attack and have been derived in Ref. 1. The numerical values of these parameters are provided in Ref. 3 for different values of angle of attack for a delta wing.

Defining the state vector $x = (x_1, x_2)^T = (\phi, \dot{\phi})^T$ and substituting Eq. (2) in Eq. (1), the system (1) can be written in a state-variable form as

$$\dot{x}_1 = x_2, \qquad \dot{x}_2 = f(x) + d_0 u \tag{3}$$

where $x \in \mathcal{R}^2$, the Euclidean space of dimension 2,

$$f(x) = b_0 + b_1x_1 + b_2x_2 + b_3|x_1|x_2 + b_4|x_2|x_2 + b_5x_1^3 \tag{4}$$

and the parameters b_i are given by ($i = 0, 1, \ldots, 5$)

$$b_i = \left(\rho U_\infty^2 Sb/2I_{xx}\right)a_i \tag{5}$$

A reference model is specified by the linear time-invariant differential equation

$$\dot{x}_m = A_m x_m \tag{6}$$

where $x_m = (x_{m1}, x_{m2})^T$, $\zeta > 0$, $\omega_n > 0$, and

$$A_m = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix}$$

For the choice of the damping ratio ζ and the natural frequency ω_n, A_m is a Hurwitz matrix, and the reference trajectory $x_m(t)$ starting from any nonzero initial condition tends to zero as $t \to \infty$. We are interested in the following two problems related to wing-rock motion control under different assumptions on the function $f(x)$.

Problem 1: adaptive control. Suppose that in the wing-rock dynamics (3) the parameters b_i, $i = 0, 1, \ldots, 5$, and d_0 are unknown, but the sign of d_0 is known. Derive a control law such that

$$\lim[x(t) - x_m(t)] \to 0, \qquad t \to \infty$$

Problem 2: neural control. Suppose that in the wing-rock dynamics (3) the nonlinear function $f(x)$ is completely unknown, and d_0 is an unknown constant but its sign is known. Derive a neural control law such that

$$\lim[x(t) - x_m(t)] \to 0, \qquad t \to \infty$$

III. Adaptive Control

In this section, the solution of Problem 1 will be obtained. Define the trajectory tracking error $e = (e_1, e_2)^T = ((x_1 - x_{m1}), (x_2 - x_{m2}))^T$. Then, using Eqs. (3) and (6), the error dynamics are given by

$$\dot{e} = A_m e + d_m[g(x) + d_0 u] \tag{7}$$

where $d_m = (0, 1)^T$ and

$$g(x) = b_0 + \left(b_1 + \omega_n^2\right)x_1 + (b_2 + 2\zeta\omega_n)x_2 + b_3|x_1|x_2 + b_4|x_2|x_2 + b_5x_1^3 \tag{8}$$

We choose an adaptive control law of the form

$$u = -\left[\theta_0(t) + \theta_1(t)x_1 + \theta_2(t)x_2 + \theta_3(t)|x_1|x_2 + \theta_4(t)|x_2|x_2 + \theta_5(t)x_1^3\right] \tag{9}$$

Now an adaptation law for the adjustment of the parameter vector $\theta = (\theta_0, \theta_1, \ldots, \theta_5)^T$ will be derived such that $e(t)$ asymptotically converges to zero following the approach of Ref. 15. Define

$$\theta_i^* = b_i/d_0, \qquad i = 0, 3, 4, 5$$
$$\theta_1^* = \left(b_1 + \omega_n^2\right)/d_0 \tag{10}$$
$$\theta_2^* = (b_2 + 2\zeta\omega_n)/d_0$$

Let the parameter error vector be

$$\tilde{\theta} = [(\theta_0^* - \theta_0), \ldots, (\theta_5^* - \theta_5)] \in \mathcal{R}^6 \tag{11}$$

Define a six-vector function

$$h(x) = \left[1, x_1, x_2, |x_1|x_2, |x_2|x_2, x_1^3\right]^T \tag{12}$$

Then using Eqs. (10) and (11) in the error dynamics (7) gives

$$\dot{e} = A_m e + d_m d_0 \tilde{\theta}^T h(x) \tag{13}$$

Since A_m is a Hurwitz matrix, given a positive definite symmetric matrix Q (denoted as $Q > 0$), there exists a unique matrix $P > 0$ such that the Lyapunov equation

$$A_m^T P + PA_m = -Q \tag{14}$$

is satisfied.

For the derivation of the adaptation law, the Lyapunov approach is used. To this end, we choose a Lyapunov function of the form

$$V(e, \tilde{\theta}) = e^T Pe + \tilde{\theta}^T \Gamma \tilde{\theta}|d_0| \tag{15}$$

where $\Gamma = \mathrm{diag}(\Gamma_i)$, $i = 1, \ldots, 6$, and $\Gamma_i > 0$. Here $V(e, \tilde{\theta})$ is a positive definite function of e, and $\tilde{\theta}$ and $V(0, 0) = 0$. The derivative of Eq. (15) along the trajectory of Eq. (13) is given by

$$\dot{V} = e^T\left(PA_m + A_m^T P\right)e + 2e^T Pd_m d_0 \tilde{\theta}^T h(x) + 2\tilde{\theta}^T \Gamma \dot{\tilde{\theta}}|d_0| \tag{16}$$

In view of Eq. (16), one chooses the adaptation law

$$\dot{\theta} = \mathrm{sgn}(d_0)\Gamma^{-1}e^T Pd_m h(x) \tag{17}$$

Since $\dot{\tilde{\theta}} = -\dot{\theta}$, substituting Eq. (17) in Eq. (16) gives

$$\dot{V} = -e^T Qe \le 0 \tag{18}$$

In view of Eq. (18), using an argument similar to that of Ref. 15, it follows that $e(t) \to 0$ as $t \to \infty$. Thus in the closed-loop system (3), (9), and (17), $(\phi(t), \dot{\phi}(t)) \to 0$ as $t \to \infty$ since the reference trajectory $x_m(t)$ asymptotically converges to zero.

IV. Neural Control

Now the derivation of the neural controller is considered, and it is assumed that the nonlinear function $f(x)$ is completely unknown to the designer. It is assumed that the output $\hat{g}(x)$ of an RBF network approximates the function $g(x)$ given in Eq. (8) over a sufficiently large region Ω of interest in the state space such that

$$g(x) = \hat{g}(x) + \epsilon(x)$$
$$\hat{g}(x) = \gamma_0 + \sum_{i=1}^{n} \gamma_i \psi_i(x) \tag{19}$$
$$|\epsilon(x)| = |g(x) - \hat{g}(x)| < \epsilon_0, \qquad x \in \Omega$$

The bound ϵ_0 on the approximation error can be made small by selecting a large number of neurons in the hidden layer. Here $\gamma = (\gamma_0, \gamma_1, \ldots, \gamma_n) \in \mathcal{R}^{n+1}$ is the weight vector, $\psi_i (i = 1, \ldots, n)$ are the outputs of the hidden layer, $\hat{g}(x)$ is the output of the neural network, and $x = (x_1, x_2)^T$ is the network input. Let $\psi(x) = (1, \psi_1(x), \ldots, \psi_n(x)) \in \mathcal{R}^{n+1}$. The function $\psi_i(x)$ is given by

$$\psi_i(x) = \exp\left[-\|x - c_i\|^2/\left(\sigma_i^2\right)\right], \qquad i = 1, \ldots, n \tag{20}$$

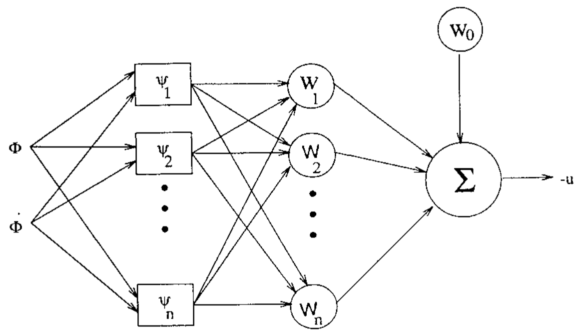


Fig. 1 RBF neural network.

where $\|\cdot\|$ denotes the Euclidean norm and c_i and $\sigma_i > 0$ are the center and width of the ith kernel unit, respectively. Here the chosen activation function is the Gaussian function, and this network is called the Gaussian RBF network. Using Eq. (19), the error dynamics (7) can be written as

$$\dot{e} = A_m e + d_m\left[\gamma^T\psi(x) + \epsilon(x) + d_0 u\right] \tag{21}$$

We choose a control law of the form

$$u = -w^T(t)\psi(x) + u_s \tag{22}$$

where the additional signal u_s is to be determined later. Here the control signal given in Eq. (22) is generated by a Gaussian RBF network and the weight vector $w(t) = (w_0, w_1, \ldots, w_n)^T \in \mathcal{R}^{n+1}$.

Define

$$w^* = \gamma/d_0, \qquad \tilde{w} = w^* - w \tag{23}$$

Substituting control law (22) in Eq. (21) and using Eq. (23) give

$$\dot{e} = A_m e + d_m d_0\left[\tilde{w}^T\psi(x) + u_s\right] + d_m\epsilon(x) \tag{24}$$

Now for the derivation of the neural control law, we choose a Lyapunov function of the form

$$V(e, \tilde{w}) = e^T P e + \tilde{w}^T\Gamma\tilde{w}\,|d_0| \tag{25}$$

where $\Gamma > 0$. Then the derivative of $V(x)$ along the solution of Eq. (24) is given by

$$\dot{V} = -e^T Q e + 2e^T P d_m\left[d_0\tilde{w}^T\psi(x) + d_0 u_s + \epsilon(x)\right] + 2\tilde{w}^T\Gamma\dot{\tilde{w}}|d_0| \tag{26}$$

In view of Eq. (24), the adaptation law is chosen as

$$\dot{w} = \mathrm{sgn}(d_0)\Gamma^{-1}e^T P d_m\psi(x) \tag{27}$$

Substituting Eq. (27) in Eq. (26) gives

$$\dot{V} = -e^T Q e + 2e^T P d_m\epsilon(x) + 2e^T P d_m d_0 u_s \tag{28}$$

$$\leq -\lambda_{\min}(Q)\|e\|^2 + |v|\epsilon_0 + v d_0 u_s \tag{29}$$

where $v = 2e^T P d_m$ and $\lambda_{\min}(Q)$ denotes the smallest eigenvalue of Q. Let d_0 be bounded as $|d_0| \geq d_{0m}$. In order to make $\dot{V}$ negative, one chooses u_s of the form

$$u_s = -k\ \mathrm{sgn}(d_0)\mathrm{sgn}(v) \tag{30}$$

where gain $k \geq \epsilon_0/d_{0m}$.

Substituting Eq. (29) in Eq. (28) gives

$$\dot{V} \leq -\lambda_{\min}(Q)\|e\|^2 \leq 0 \tag{31}$$

In view of Eq. (29), it follows that $e(t) = [(\phi(t) - x_{m1}(t)), (\dot{\phi}(t) - x_{m2}(t))]^T$ converges to zero. Since $x_{mi}(t) \to 0$, it follows that $\phi(t) \to 0, \dot{\phi}(t) \to 0$, as $t \to \infty$.

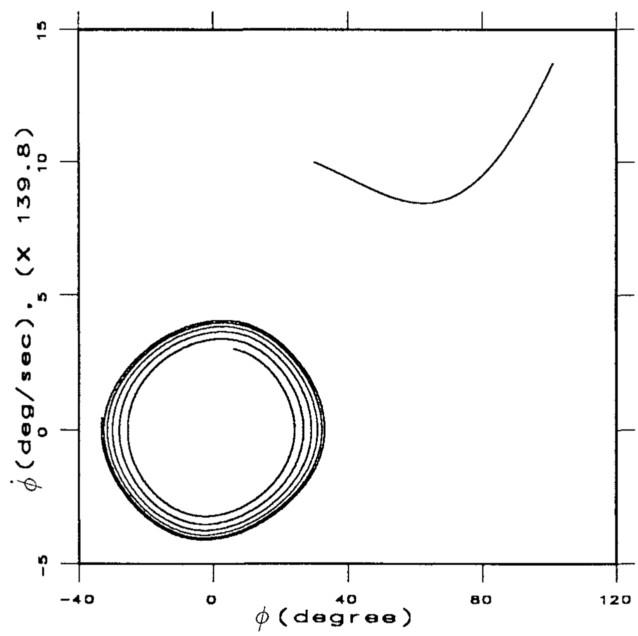


Fig. 2 Phase-plane portrait of open-loop system.

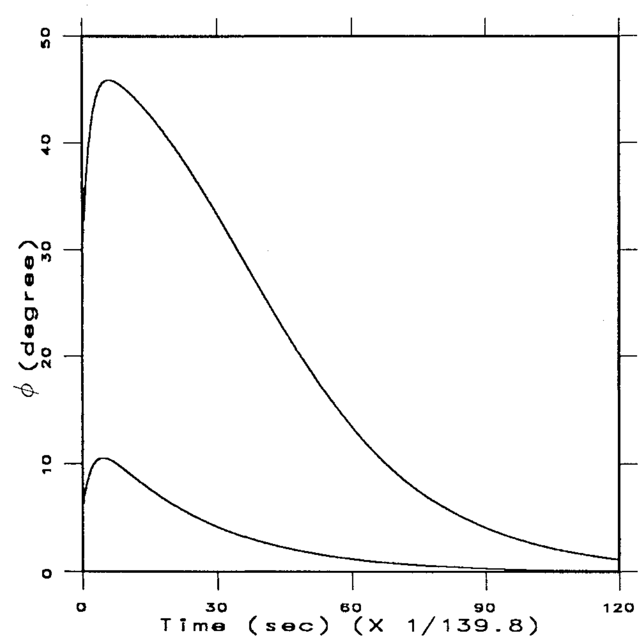


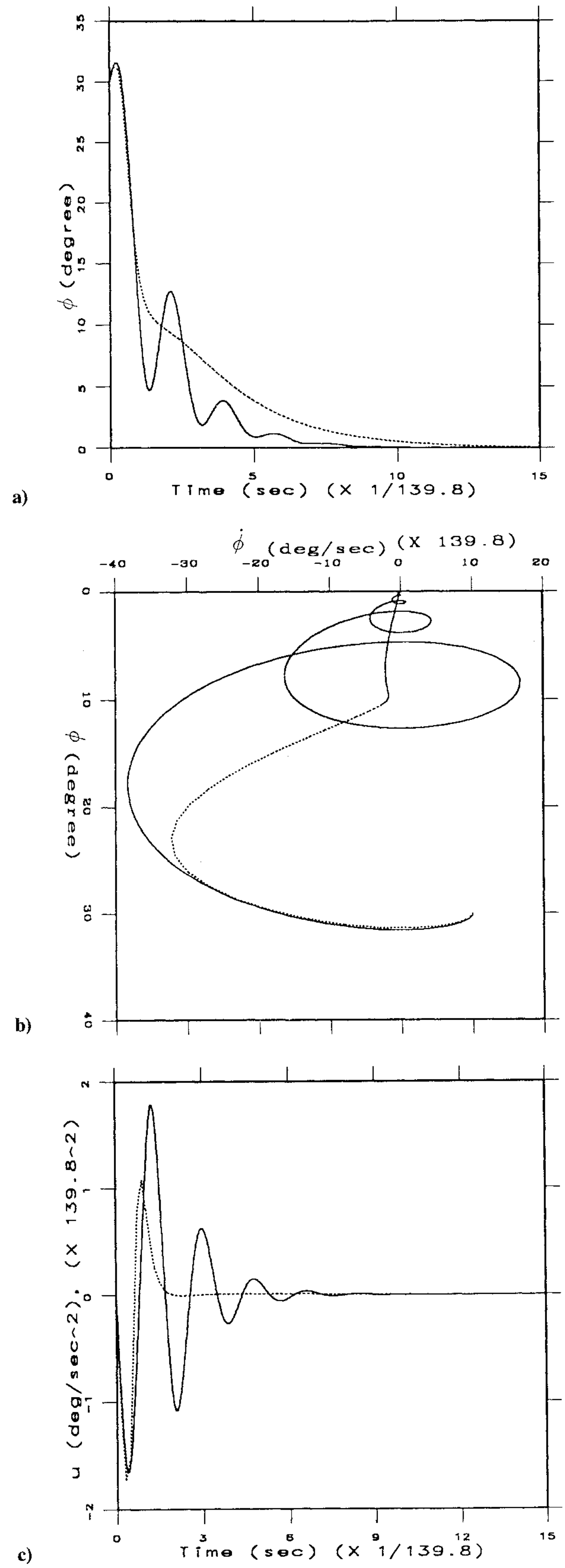
Fig. 3 Roll-rate feedback control.

Usually, when one selects a RBF network with a sufficiently large number of neurons such that ϵ is negligible, since $\epsilon_0 = 0$, one sets $u_s = 0$ in Eq. (22), and using a simplified control law

$$u = -w^T(t)\psi(x) \tag{32}$$

one can show that $\dot{V}$ satisfies Eq. (31) and, therefore, $(\phi(t), \dot{\phi}(t))$ converges to zero.

Figure 1 shows the neural network generating the control input given by Eq. (32). It is interesting to note that if the weight vector is $w = w^* = (\gamma/d_0)$, then the neural network's output $-u$ given in Eq. (32) is $(w^*)^T\psi(x) = \hat{g}(x)/d_0$. Thus, when w converges to w^*, the network's output approximates the nonlinear function $g(x)/d_0$. Since $f(x) - g(x)$ is a known function, one can obtain an approximation of the nonlinear function $f(x)$ in the wing model if an estimate of the control effectiveness element d_0 is known. It is pointed out that the convergence of the tracking error e to zero does not, in general, imply that the weight vector w converges to its correct value unless the signal $\psi(x)$ in the adaptation law (27) is persistently exciting.[11,15]



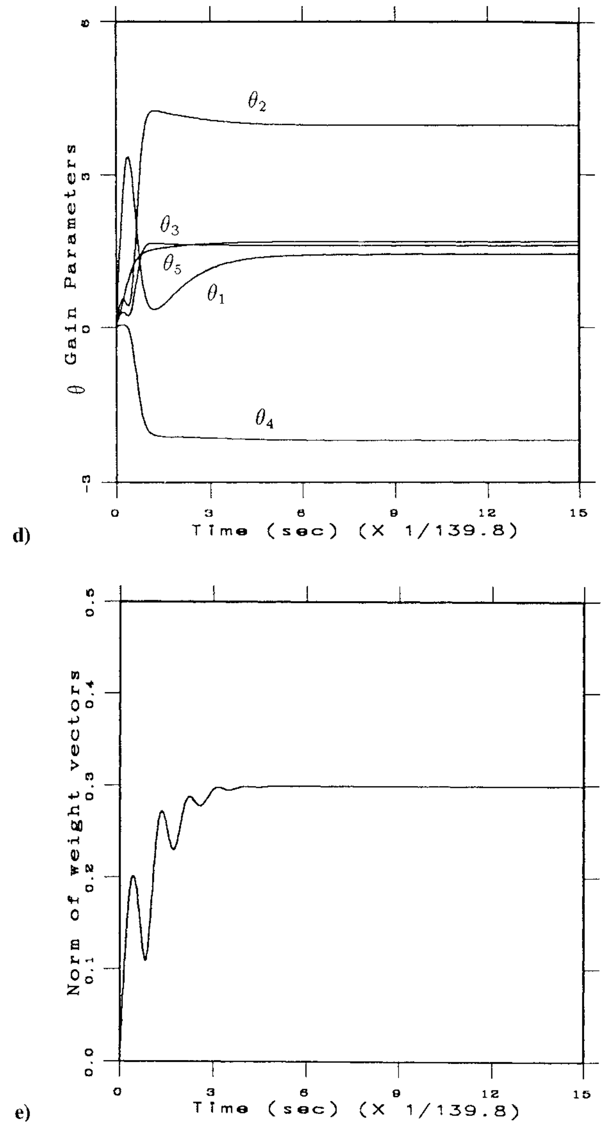


Fig. 4 Adaptive and neural control for large initial condition (dotted lines, adaptive control; solid lines, neural control): a) angular position ϕ, b) angular rate $\dot{\phi}$ and angular position ϕ, c) control input u, d) gain parameters θ_i ($i = 1, \ldots, 5$), and e) norm of neural network weight vector w.

V. Simulation Results

In this section, the results of digital simulation for a wing model given in Ref. 3 are presented. The aerodynamic parameters of the delta wing for 25 deg angle of attack are used for simulation. It is assumed that $U_\infty = 15$ m/s and $b = 0.429$ m. Dimensionless time t^*, where $t^* = (4U_\infty/b)t$, has been introduced as an independent variable in Refs. 2 and 3 for obtaining the roll dynamics. The parameters b_i for the model (3) with t^* as an independent variable in which $\dot{x}_i$ is replaced by x_i' (where the prime denotes derivatives with respect to t^*) are given by

$$b_0 = 0, \qquad b_1 = -0.01859521, \qquad b_2 = 0.015162375,$$
$$b_3 = -0.06245153, \qquad b_4 = 0.00954708,$$
$$b_5 = 0.02145291, \qquad d_0 = 1$$

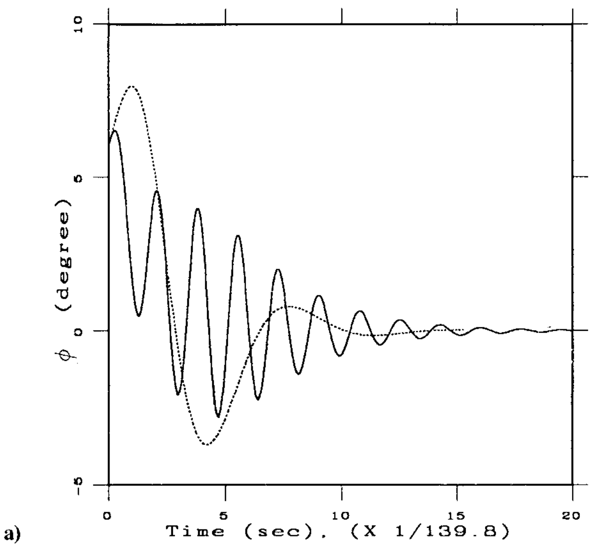
For simplicity, the reference trajectory is chosen as $x_m(t) = 0$. The parameters ζ and ω_n of matrix A_m are 0.707 and 0.5, respectively. For a choice of $Q = -I$, solving the Lyapunov equation (12) gives

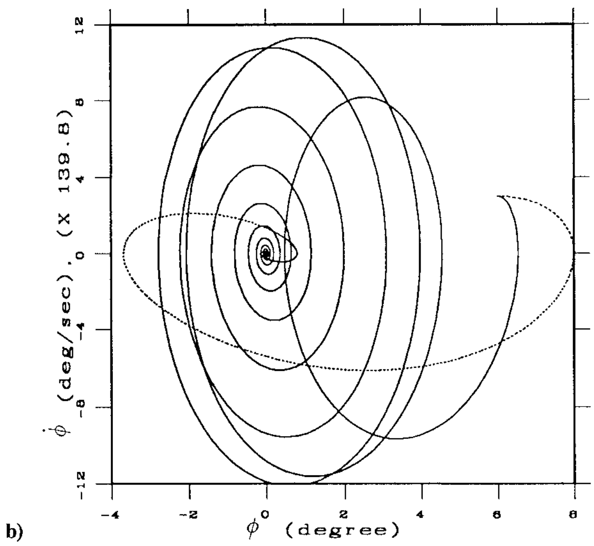
$$p_{12} = 1/\left(2\omega_n^2\right)$$
$$p_{22} = (0.5 + p_{12})/2\zeta\omega_n$$
$$p_{11} = 2\zeta\omega_n p_{12} + \omega_n^2 p_{22}$$

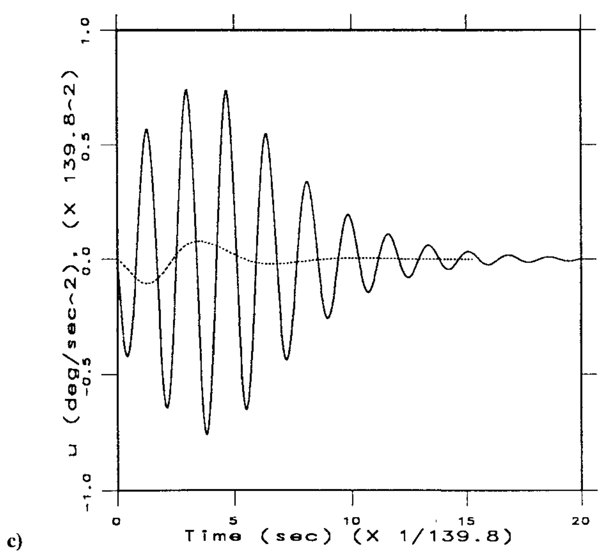
where the matrix $P = (p_{ij}), i, j = 1, 2$, and I denotes the identity matrix of appropriate dimension.

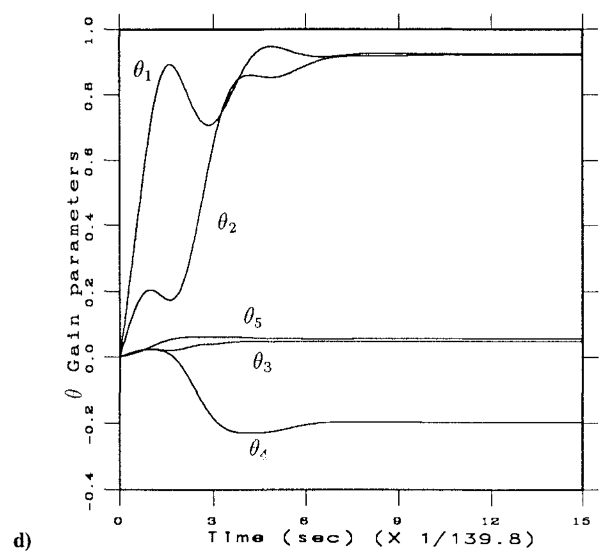
A. Open-Loop Dynamics and Roll Rate Feedback

The open-loop system (3) with $u = 0$ was simulated for two initial conditions: $\phi(0) = 6$ deg, $\dot{\phi}(0) = 419.4$ deg s^{-1} and $\phi(0) = 30$









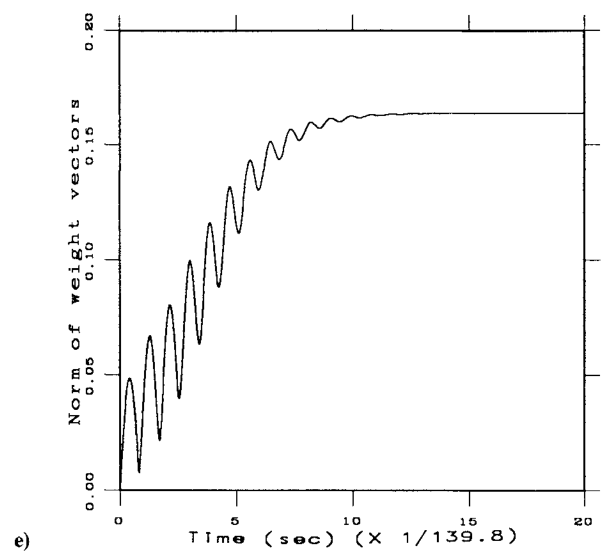


Fig. 5 Adaptive and neural control for small initial condition (dotted lines, adaptive control; solid lines, neural control): a) angular position ϕ, b) angular rate $\dot{\phi}$ and angular position ϕ, c) control input u, d) gain parameters θ_i ($i = 1, \ldots, 5$), and e) norm of neural network weight vector w.

deg, $\dot{\phi}(0) = 1398 \deg \mathrm{s}^{-1}$. The phase-plane plot is shown in Fig. 2. For a smaller initial condition a limit cycle is obtained, and roll-angle divergence takes place when the initial condition is larger.

A simple fixed-gain roll-rate feedback controller with $u = -Kp$ was simulated. For small gains K, the system was unstable, and sluggish responses were obtained for large values of K. After several trials gain $K = 0.5$ was chosen to obtain good responses (Fig. 3). For a larger initial condition, roll-angle response took relatively longer time for convergence.

B. Adaptive Control

To examine the performance of the adaptive controller, the complete closed-loop system (3), (9), and (17) was simulated for the two initial conditions given in Sec. V.A. Since $b_0 = 0$, for simplicity, we have set $\theta_0(t) = 0$ in the control law (9). The initial condition for gain parameters was assumed to be $\theta(0) = 0$. The weighting matrix was taken as $\Gamma = \frac{1}{15} I$. The responses are shown (by dotted lines) in Figs. 4 and 5 for larger and smaller initial conditions, respectively. We observe smooth convergence of the roll angle to zero, and the gains $\theta_i(t)$ of the adaptive controller converge to constant values, as shown in Fig. 4d. For the chosen controller parameters, the required control input is shown in Figs. 4c and 5c. As expected for the larger initial condition the control input is relatively larger.

C. Neural Control

A RBF network of 441 neurons in a hidden layer was used for simulation. The centers c_i ($i = 1, \ldots, 441$) of the Gaussian functions were uniformly spaced in the ϕ-ϕ' plane such that for an appropriate choice of j_1 and j_2

$$c_i = \begin{bmatrix} j_1\Delta_1 \\ j_2\Delta_2 \end{bmatrix}$$

where $\Delta_1 = 0.2$ rad, $\Delta_2 = 0.1$ rad, and $j_1, j_2 \in \{-10, -9, \ldots, 9, 10\}$. The widths σ_i of the kernel units were set to 1. The weighting matrix was taken as $\Gamma = 0.2I$. The complete closed-loop system of Eqs. (3) and (27) and the simplified neural controller of Eq. (32) was simulated. The responses for the larger and smaller initial conditions are shown as solid lines in Figs. 4 and 5, respectively. Since sufficient numbers of neurons were used in the network, additional signal u_s given in Eq. (30) was not necessary for control, and we have set $u_s = 0$. The initial value of the weight vector of the neural network was set to $w(0) = 0$. We have taken a zero-bias term in the output of the network by setting the first element $w_0(t) = 0$. We observe that compared to the adaptive controller, the neural controller gives rise to oscillatory responses. This should be of no surprise since the adaptive controller has additional information related to the structure of nonlinearity in the model, whereas the neural control tries to learn the nonlinear function in the transient period. However, the roll angle converges to zero in spite of the unknown dynamics of the system. The norm $||w||$ of the weight vector w is plotted in Figs. 4e and 5e.

Remark 1. Comparing the roll-angle response obtained using the fixed-gain controller of Fig. 3 to the responses obtained using the adaptive and neural controller, we observe that, although simple roll-rate feedback gives smooth responses, the response time is more than 10 times large for the larger initial condition. Moreover, tuning was done in several trials to obtain the final value of the roll-rate feedback gain for obtaining good responses. It should be noted that tuning of gain K cannot be done when the dynamic model is unknown. Furthermore, since the aerodynamic parameters depend on, e.g., the dynamic pressure and the angle of attack, it is difficult to predict the performance for different flight conditions using a fixed-gain controller. The advantage of the neural controller over the adaptive and any fixed-gain controller lies in its ability to learn nonlinear maps. However, when the structure of the roll dynamics is known, the adaptive controller is preferable. Interestingly, the adaptive and the neural controller adapt themselves to compensate for the uncertain aerodynamic parameters and nonlinear functions, respectively, in the highly nonlinear model.

Remark 2. We have also simulated the wing-rock dynamic model of Luo and Lan[7] (see also Refs. 16 and 17) including the adaptive and the neural controller. We observed that in the closed-loop system roll angle is smoothly controlled. Interested readers may find these results in Ref. 18.

VI. Conclusions

Adaptive and neural control of wing-rock motion of a slender delta wing was considered. Two kinds of roll dynamics were considered in the design. The first model was characterized by a linearly parameterized rolling moment, and for the second model, no knowledge of the roll dynamics was assumed. Based on the Lyapunov stability theory, an adaptive controller for the first model and a neural controller for the second model were derived. The neural controller includes a Gaussian radial basis function network in the feedback path. Simulation results were presented to show that in the closed-loop system wing-rock control can be accomplished using the adaptive and neural controller in spite of the uncertainty in the roll dynamics.

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